What Is Mathematics?

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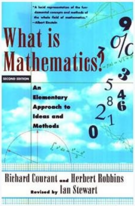
(Redirected from What Is Mathematics)

What Is Mathematics? is a mathematics book written by Richard Courant and Herbert Robbins, published in England by Oxford University Press. It is an introduction to mathematics, intended both for the mathematics student and for the general public.

First published in 1941, it discusses number theory, geometry, topology and calculus. A second edition was published in 1996 with an additional chapter on recent progress in mathematics, written by Ian Stewart.

What is Mathematics



Cover of 1996 second edition

Author	Richard Courant and Herbert Robbins
Language	English
Subject	Mathematics
Publisher	Oxford University Press
Publication date	1941
ISBN	0-19-502517-2
OCLC	16608993

Contents [hide]

Authorship [edit]

The book was based on Courant's course material. Although Robbins assisted in writing a large part of the book, he had to fight for authorship. Nevertheless, Courant alone held the copyright for the book. This resulted in Robbins receiving a smaller share of the royalties.[1][2]

Title [edit]

Michael Katehakis remembers Robbins' interest in the literature and Tolstoy in particular and he is convinced that the title of the book is most likely due to Robbins, who was inspired by the title of the essay What is Art? by Leo Tolstoy. Robbins did the same in the book *Great Expectations : The Theory of Optimal Stopping* he co-authored with Yuan-Shih Chow and David Siegmund, where one can not miss the connection with the title of the novel *Great Expectations* by Charles Dickens.

According to Constance Reid,[2] Courant finalized the title after a conversation with Thomas Mann.

Translations [edit]

- The first Russian translation «Что такое математика?» was published in 1947; there were 5 translations since then, the last one in 2010.
- The first Italian translation, *Che cos'è la matematica?*, was published in 1950. A translation of the second edition was issued in 2000.
- The first German translation "Was ist Mathematik?" was published in 1962.
- A Spanish translation of the second edition, *¿Qué Son Las Matemáticas?*, was published in 2002.

Reviews [edit]

- What Is Mathematics? An Elementary Approach to Ideas and Methods, book review by Brian E. Blank, *Notices of the American Mathematical Society* **48**, #11 (December 2001), pp. 1325–1330
- What Is Mathematics?, book review by Leonard Gillman, *The American Mathematical Monthly* **105**, #5 (May 1998), pp. 485–488.

Editions [edit]

- Richard Courant and Herbert Robbins (1941). *What is Mathematics?: An Elementary Approach to Ideas and Methods*. London: Oxford University Press. ISBN 0-19-502517-2.
- (1996) 2nd edition, with additional material by Ian Stewart. New York: Oxford University Press. ISBN 0-19-510519-2.
- Courant, Richard; Robbins, Herbert (2015). *Qu'est-ce que les mathématiques ? Une introduction élémentaire aux idées et aux méthodes*. Cassini. ISBN 9782842252045. French translation of the second English edition by Marie Anglade and Karine Py.
- Courant, Richard; Robbins, Herbert; Stewart, Ian (2002). *¿Qué Son Las Matemáticas? Conceptos y métodos fundamentales* (in Spanish). México, D. F.: Fondo de Cultura Económica. ISBN 968-16-6717-4. Spanish translation of the second English edition.
- Courant, Richard; Robbins, Herbert (1950). *Che cos'è la matematica? Introduzione elementare ai suoi concetti e metodi* (in Italian). Turin: Einaudi. (first Italian translation, from the 1945 English edition)
- Courant, Richard; Robbins, Herbert (1971). *Che cos'è la matematica? Introduzione elementare ai suoi concetti e metodi* (in Italian). Turin: Boringhieri. (based on the previous Eianudi's edition)
- Courant, Richard; Robbins, Herbert (1984). *Toán học là gì* (in Vietnamese). Hanoi: Khoa học Kỹ thuật. (Vietnamese translation by Hàn Liên Hải from the Russian edition)
- Courant, Richard; Robbins, Herbert; Stewart, Ian (2000). *Che cos'è la matematica? Introduzione elementare ai suoi concetti e metodi* (in Italian). Turin: Bollati Boringhieri. ISBN 88-339-1200-0. (Italian translation of the second English edition)

References [edit]

1. ^ Page, Warren; Robbins, Herbert (1984), "An Interview with Herbert Robbins", *The College Mathematical Journal*, The Mathematical Association of America, **15** (1): 5, doi:10.2307/3027425, JSTOR 3027425
2. ^ *a* *b* Reid, Constance, Courant in Göttingen and New York. The story of an improbable mathematician. Springer-Verlag, New York-Heidelberg, 1976. ii+314 pp.

- Herbert Robbins, *Great Expectations: The Theory of Optimal Stopping*, with Y. S. Chow and David Siegmund. Boston: Houghton Mifflin, 1971.

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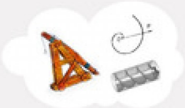
Isaac Newton

English physicist and mathematician who is widely regarded as one of the most influential scientists of all time and as a key figure in the scientific revolution. His book Philosophiae Naturalis Principia Mathematica, first published in 1687, laid the foundations for most of classical mechanics. Newton also made seminal contributions to optics and shares credit for the invention of the infinitesimal calculus.

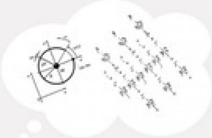


Euclid

Euclid was a Greek mathemetician, who is frequently cited as the "Father of Geometry." His work "Elements" is one of the most influential works in the history of mathematics, and described the principles of Euclidean geometry.

Archimedes

Archimedes is universally acknowledged to be the greatest of ancient mathematicians. He used the method of exhaustion to calculate the area under the arc of a parabola with the summation of an infinite series, and gave a remarkably accurate approximation of pi.

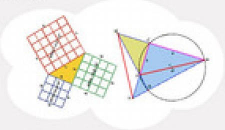


Gauss

was known as the 'Prince of Mathematicians.' He contributed in many ways to math and science. Some of his notable inventions include a device called the heliotrope and the first electric telegraph. He also discovered the orbit of the asteroid Ceres.

Pythagoras

who is sometimes called the "First Philosopher," studied under Anaximander, Egyptians and Babylonians became the most influential of early Greek mathematicians. He is credited with being first to use axioms and deductive proofs and his influence on Plato and Euclid is enormous.

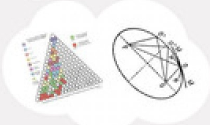


Lagrange

He made significant contributions to all fields of analysis, number theory, and both classical and celestial echanics. Offered the most comprehensive treatment of classical mechanics since Newton and formed a basis for the development of mathematical physics in the nineteenth century.

Pierre de Fermat

made notable contributions to analytic geometry, probability, and optics. He is best known for Fermat's Last Theorem, which he described in a note at the margin of a copy of Diophantus'Arithmetica. He also is given credit for early developments that led to infinitesimal calculus, including his technique ofadequality.

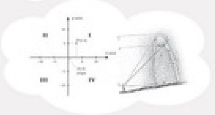


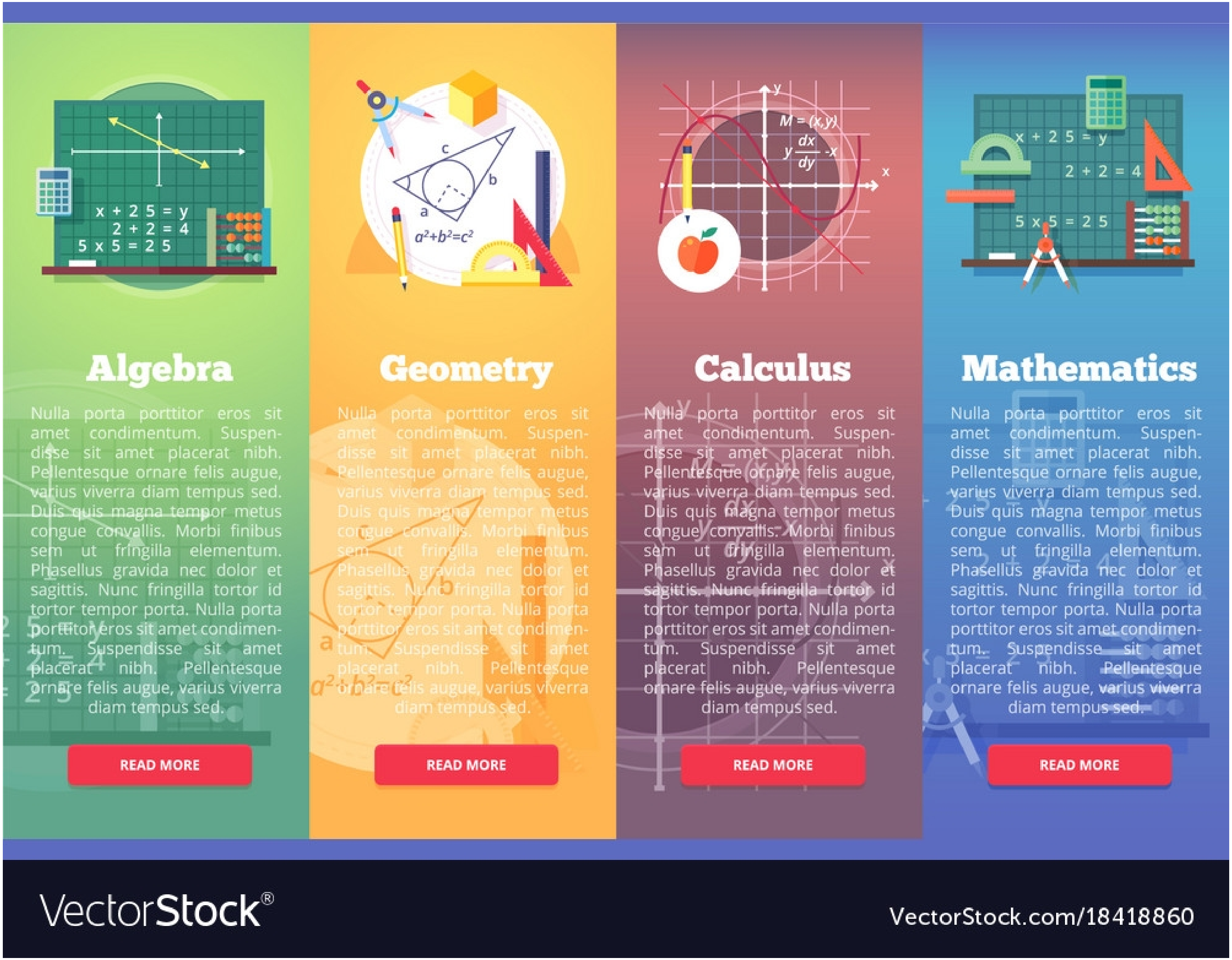
René Déscartes

has been dubbed "The Father of Modern Philosophy", and much subsequent Western philosophy is a response to his writings. But Descartes's influence in mathematics is equally apparent; He is credited as the father of analytical geometry, the bridge between algebra and geometry, crucial to the discovery ofinfinitesimal calculus and analysis.Also, the Cartesian coordinate system — allowing algebraic equations to be expressed as geometric shapes in a two-dimensional coordinate system was named after him.

Leibniz

German mathematician and philosopher. He occupies a prominent place in thehistory of mathematics and was the first to describe a pinwheel calculator, also invented the Leibniz wheel, used in the arithmometer, the first mass-produced mechanical calculator. He also refined the binary number system, which is at the foundation of virtually all digital computers.

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a²+b²=c²
Geometry
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y dx/dy -x
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Mathematics
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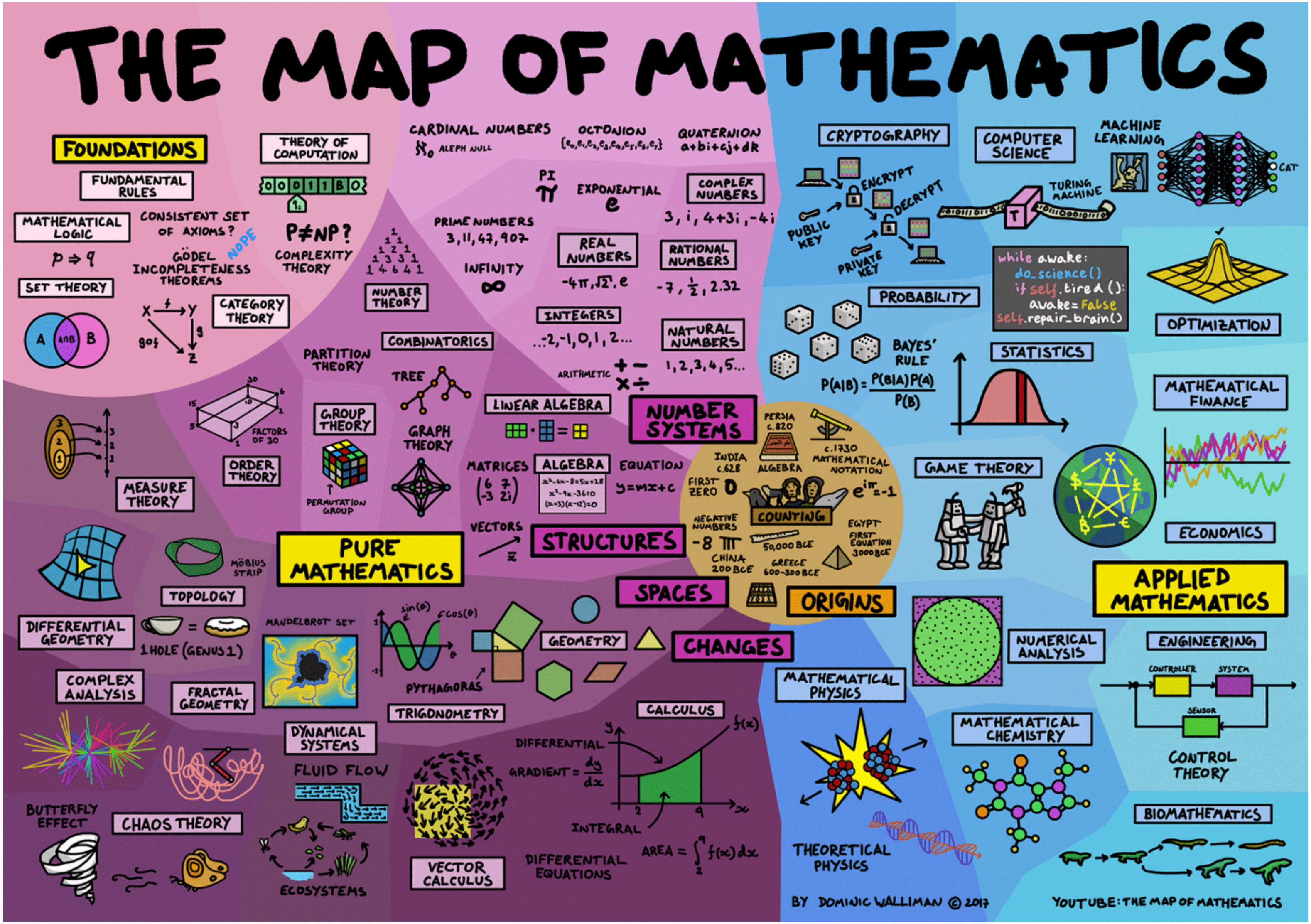
THE MAP OF MATHEMATICS
FOUNDATIONS
FUNDAMENTAL RULES
MATHEMATICAL LOGIC
p ⇒ q
CONSISTENT SET OF AXIOMS?
NOPE
GÖDEL INCOMPLETENESS THEOREMS
SET THEORY
A
A∩B
B
CATEGORY THEORY
X
f
Y
g∘f
g
Z
THEORY OF COMPUTATION
P≠NP?
COMPLEXITY THEORY
NUMBER THEORY
CARDINAL NUMBERS
ℵ₀ ALEPH NULL
OCTONION
QUATERNION
a+bi+cj+dk
PI
π
EXPONENTIAL
e
COMPLEX NUMBERS
3, i, 4+3i, -4i
PRIME NUMBERS
3,11,47,907
INFINITY
∞
REAL NUMBERS
-4π, √2, e
RATIONAL NUMBERS
-7, 1/2, 2.32
INTEGERS
...-2,-1,0,1,2...
NATURAL NUMBERS
1,2,3,4,5...
ARITHMETIC
+ − × ÷
COMBINATORICS
PARTITION THEORY
TREE
GROUP THEORY
GRAPH THEORY
PERMUTATION GROUP
LINEAR ALGEBRA
NUMBER SYSTEMS
MATRICES
ALGEBRA
EQUATION
y=mx+c
VECTORS
STRUCTURES
PURE MATHEMATICS
SPACES
CHANGES
ORIGINS
PERSIA c.820
ALGEBRA
INDIA c.628
FIRST ZERO
0
c.1730 MATHEMATICAL NOTATION
e^{iπ}=-1
COUNTING
50,000 BCE
NEGATIVE NUMBERS
-8
CHINA 200 BCE
GREECE 600-300 BCE
EGYPT FIRST EQUATION 3000 BCE
FACTORS OF 30
ORDER THEORY
MEASURE THEORY
TOPOLOGY
MÖBIUS STRIP
DIFFERENTIAL GEOMETRY
1 HOLE (GENUS 1)
MANDELBROT SET
COMPLEX ANALYSIS
FRACTAL GEOMETRY
sin(θ)
cos(θ)
PYTHAGORAS
TRIGONOMETRY
GEOMETRY
CALCULUS
f(x)
DIFFERENTIAL
GRADIENT = dy/dx
INTEGRAL
AREA = ∫₂⁹ f(x) dx
DIFFERENTIAL EQUATIONS
DYNAMICAL SYSTEMS
FLUID FLOW
ECOSYSTEMS
VECTOR CALCULUS
BUTTERFLY EFFECT
CHAOS THEORY
CRYPTOGRAPHY
ENCRYPT
DECRYPT
PUBLIC KEY
PRIVATE KEY
COMPUTER SCIENCE
TURING MACHINE
MACHINE LEARNING
CAT
while awake:
do_science()
if self.tired():
awake = False
self.repair_brain()
OPTIMIZATION
PROBABILITY
BAYES' RULE
P(A|B) = P(B|A)P(A) / P(B)
STATISTICS
MATHEMATICAL FINANCE
GAME THEORY
ECONOMICS
APPLIED MATHEMATICS
NUMERICAL ANALYSIS
ENGINEERING
CONTROLLER
SYSTEM
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Differential Equations

Review of the Indefinite Integral

The function $F(x)$ is called an **antiderivative** of $f(x)$ if $F'(x) = f(x)$.

EX: $F(x) = 2x^3$ is an antiderivative of $f(x) = 6x^2$ because $\frac{d}{dx}(2x^3) = 6x^2$. Similarly, $F(x) = 2x^3 + 7$ is also an antiderivative of $f(x) = 6x^2$ because $\frac{d}{dx}(2x^3 + 7) = 6x^2$.

In general, if $F(x)$ is an antiderivative of $f(x)$, then $F(x) + C$, where C is a constant, is also an antiderivative of $f(x)$.

The symbol $\int f(x)\,dx$ is used to represent any antiderivative of $f(x)$. In this notation, $f(x)$ is called the **integrand**. An antiderivative $\int f(x)\,dx$ is also called an **indefinite integral**.

Review of Integration

- $\int 0\,dx = C$, for some constant C
- $\int 1\,dx = x + C$
- $\int k\,dx = kx + C$, where k is a constant
- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, for any rational number n, where $n \neq -1$
  - $\int \frac{1}{x}\,dx = \ln|x| + C$
  - $\int e^x\,dx = e^x + C$
- $\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$, where k is a constant
- $\int \sin x\,dx = -\cos x + C$
- $\int \cos x\,dx = \sin x + C$
- $\int \tan x\,dx = -\ln|\cos x| + C$
- $\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$
- $\int [f(x) - g(x)]\,dx = \int f(x)\,dx - \int g(x)\,dx$
- $\int kf(x)\,dx = k\int f(x)\,dx$, where k is a constant

To perform integration by parts:

If $u(x)$ and $v(x)$ are functions, the product rule of differentiation yields $\frac{d}{dx}(uv) = uv' + vu'$. To use integration by parts, follow these steps to undo the product rule.

Step 1: Factor the integrand into two parts, u and dv, so that the integral appears as $\int u\,dv$.

Step 2: Use differentiation to find du, and integrate dv to find v.

Step 3: Apply the rule $\int u\,dv = uv - \int v\,du$.

Step 4: Find $\int v\,du$ to complete the integration.

To perform integration by substitution:

To find an integral of the form $\int f(g(x))g'(x)\,dx$, use substitution to undo the chain rule of differentiation.

Step 1: Set $u = g(x)$, where $g(x)$ is chosen so as to simplify the integrand.

Step 2: Substitute $u = g(x)$ and $du = g'(x)$ into the integrand. (**NOTE:** This step usually requires multiplying or dividing by a constant.)

Step 3: Integrate to find the antiderivative $\int f(u)\,du = F(u) + C$.

Step 4: Substitute $u = g(x)$ to rewrite the antiderivative in the form $F(g(x)) + C$.

Basic Definitions

A **differential equation** is an equation involving an unknown function and one or more of its derivatives.

EX: The following equations are differential equations.

- $y' = 2x + y + 3$
- $\frac{dy}{dx} - 2y = e^x$
- $-2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 5xy$
- $\frac{\partial^2 u}{\partial x^2} = -\frac{\partial^2 u}{\partial t^2}$

Solutions of a Differential Equation

- A **solution** of a differential equation is a function such that the derivatives of the function, the independent variables, and the dependent variable all satisfy the original equation. A differential equation can have one unique solution, no solution, or infinitely many solutions.
- In an **explicit solution**, the dependent variable can be expressed solely in terms of the independent variable and constants.

  EX: $y = 3x^2$ is in the form of an explicit solution.
- In an **implicit solution**, the dependent variable is not expressed solely in terms of the independent variable and constants. The solution function is an implicit function.

  EX: $x^2 + y^2 - 25 = 0$ is in the form of an implicit solution.
- The **trivial solution** is the function $y = 0$.
- A **general solution** of a differential equation is a function that contains arbitrary constants.

  EX: $y = \sqrt{c - x^2}$ is in the form of a general solution, where c is a constant.
- A **particular solution** of a differential equation is a function that is free of all arbitrary constants.

  EX: $y = \sqrt{16 - x^2}$ is in the form of a particular solution.

Verifying a Solution of a Differential Equation

You can verify that a function is a solution of a differential equation by substituting the function and its derivatives into the equation and confirming that the result is an identity.

EX: Verify that the function $y = \sqrt{16 - x^2}$ is a solution of the differential equation $\frac{dy}{dx} + \frac{x}{y} = 0$.

a. $\frac{dy}{dx} + \frac{x}{y} = 0$ Original differential equation

b. $\frac{dy}{dx} = \frac{1}{2}(16 - x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{16 - x^2}}$ This is the derivative of the given solution function.

c. $\frac{-x}{\sqrt{16 - x^2}} + \frac{x}{\sqrt{16 - x^2}} = 0$ Substitute y, y', and y' into the equation $\frac{dy}{dx} + \frac{x}{y} = 0$.

d. $0 = 0$ Simplify.

The result is the identity $0 = 0$, so the function $y = \sqrt{16 - x^2}$ is a solution of the differential equation.

Classifying Differential Equations

Classification by Type

An **ordinary differential equation (ODE)** is an equation that contains only ordinary derivatives of one or more dependent variables.

EX: The following equations are ODEs.

- $y' + 5y = -2x$
- $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$
- $y'' + y' - 8y = 0$

A **partial differential equation (PDE)** is an equation that contains the partial derivatives of one or more dependent variables with respect to two or more independent variables.

EX: The following equations are PDEs.

- $\frac{\partial^2 u}{\partial t^2} = 100\frac{\partial^2 u}{\partial x^2}$
- $\frac{\partial u}{\partial t} = -0.25\frac{\partial^2 u}{\partial x^2}$
- $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2xy$

Classification by Order

The **order** of a differential equation is the order of the highest derivative in the equation.

EX:

- $y' + 5y = -2x$ is a first-order differential equation.
- $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$ is a second-order ODE.
- $\frac{\partial u}{\partial t} = -0.25\frac{\partial^2 u}{\partial x^2}$ is a second-order PDE.

Classification by Linearity

Assume that a differential equation can be written in the form $y^{(n)} = f(x, y, y', \ldots, y^{(n-1)})$, where $y^{(n)}$ is the highest-order derivative and f is a function of the independent variable, dependent variable, and lower-order derivatives.

A **linear differential equation** is an equation in which f is a linear function of $y, y', y'', \ldots, y^{(n-1)}$. That is, the differential equation can be written in the form $b_n(x)y^{(n)} + b_{n-1}(x)y^{(n-1)} + \ldots + b_1(x)y' + b_0(x)y = g(x)$.

EX:

- $x^2y'' + \sin(x)y = x^4$ is linear because each coefficient of y or one of its derivatives is a function of x.
- $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$ is also linear.

If an equation contains functions of y such as y^2 or functions of the derivatives of y such as $\sin(y')$, then the differential equation is **nonlinear**.

EX:

- $y'' + xyy' + y' = 2x$ is nonlinear because the coefficient of y' is a function of y.
- $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + xy = 0$ is nonlinear because the power of $\frac{dy}{dx}$ is not 1.
- $(3y)y'' + (1 - x)y' + y = 10x$ is nonlinear because the coefficient of y'' depends on y.

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THEORY

DERIVATIVES AND DIFFERENTIATION

Definition: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

DERIVATIVE RULES

1. **Sum and Difference:** $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$
2. **Scalar Multiple:** $\frac{d}{dx}(cf(x)) = cf'(x)$
3. **Product:** $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

   Mnemonic: If f is "hi" and g is "ho," then the product rule is "ho d hi plus hi d ho."
4. **Quotient:** $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$

   Mnemonic: "Ho d hi minus hi d ho over ho ho."
5. **The Chain Rule**
   - First formulation: $(f \circ g)'(x) = f'(g(x))\,g'(x)$
   - Second formulation: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
6. **Implicit differentiation:** Used for curves when it is difficult to express y as a function of x. Differentiate both sides of the equation with respect to x. Use the chain rule carefully whenever y appears. Then, rewrite $\frac{dy}{dx} = y'$ and solve for y'.

   Ex: $x\cos y - y^2 = 3x$. Differentiate to first obtain $\frac{dx}{dx}\cos y + x\frac{d(\cos y)}{dx} - 2y\frac{dy}{dx} = 3\frac{dx}{dx}$, and then $\cos y - x(\sin y)y' - 2yy' = 3$. Finally, solve for $y' = \frac{\cos y - 3}{x\sin y + 2y}$.

COMMON DERIVATIVES

1. **Constants:** $\frac{d}{dx}(c) = 0$
2. **Linear:** $\frac{d}{dx}(mx+b) = m$
3. **Powers:** $\frac{d}{dx}(x^n) = nx^{n-1}$ (true for all real $n \neq 0$)
4. **Polynomials:** $\frac{d}{dx}(a_nx^n + \cdots + a_2x^2 + a_1x + a_0) = a_nnx^{n-1} + \cdots + 2a_2x + a_1$
5. **Exponential**
   - **Base e:** $\frac{d}{dx}(e^x) = e^x$
   - **Arbitrary base:** $\frac{d}{dx}(a^x) = a^x \ln a$
6. **Logarithmic**
   - **Base e:** $\frac{d}{dx}(\ln x) = \frac{1}{x}$
   - **Arbitrary base:** $\frac{d}{dx}(\log_a x) = \frac{1}{x\ln a}$
7. **Trigonometric**
   - **Sine:** $\frac{d}{dx}(\sin x) = \cos x$
   - **Cosine:** $\frac{d}{dx}(\cos x) = -\sin x$
   - **Tangent:** $\frac{d}{dx}(\tan x) = \sec^2 x$
   - **Cotangent:** $\frac{d}{dx}(\cot x) = -\csc^2 x$
   - **Secant:** $\frac{d}{dx}(\sec x) = \sec x \tan x$
   - **Cosecant:** $\frac{d}{dx}(\csc x) = -\csc x \cot x$
8. **Inverse Trigonometric**
   - **Arcsine:** $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
   - **Arccosine:** $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
   - **Arctangent:** $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
   - **Arccotangent:** $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$
   - **Arcsecant:** $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
   - **Arccosecant:** $\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

INTEGRALS AND INTEGRATION

DEFINITE INTEGRAL

The **definite integral** $\int_a^b f(x)\,dx$ is the **signed area** between the function $y = f(x)$ and the x-axis from $x = a$ to $x = b$.

- **Formal definition:** Let n be an integer and $\Delta x = \frac{b-a}{n}$. For each $k = 0, 2, \ldots, n-1$, pick point x_k^* in the interval $[a + k\Delta x, a + (k+1)\Delta x]$. The expression $\Delta x \sum_{k=0}^{n-1} f(x_k^*)$ is a **Riemann sum**. The definite integral $\int_a^b f(x)\,dx$ is defined as $\lim_{n\to\infty} \Delta x \sum_{k=0}^{n-1} f(x_k^*)$.

INDEFINITE INTEGRAL

- **Antiderivative:** The function $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$.
- **Indefinite integral:** The indefinite integral $\int f(x)\,dx$ represents a **family** of antiderivatives: $\int f(x)\,dx = F(x) + C$ if $F'(x) = f(x)$.

FUNDAMENTAL THEOREM OF CALCULUS

Part 1: If $f(x)$ is continuous on the interval $[a, b]$, then the area function $F(x) = \int_a^x f(t)\,dt$ is continuous and differentiable on the interval and $F'(x) = f(x)$.

Part 2: If $f(x)$ is continuous on the interval $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

APPROXIMATING DEFINITE INTEGRALS

1. **Left-hand rectangle approximation:**
$$L_n = \Delta x \sum_{k=0}^{n-1} f(x_k)$$
2. **Right-hand rectangle approximation:**
$$R_n = \Delta x \sum_{k=1}^{n} f(x_k)$$
3. **Midpoint Rule:**
$$M_n = \Delta x \sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)$$
4. **Trapezoidal Rule:** $T_n = \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n))$
5. **Simpson's Rule:** $S_n = \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))$

TECHNIQUES OF INTEGRATION

1. **Properties of Integrals**
   - **Sums and differences:** $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
   - **Constant multiples:** $\int cf(x)\,dx = c\int f(x)\,dx$
   - **Definite integrals: reversing the limits:** $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
   - **Definite integrals: concatenation:** $\int_a^p f(x)\,dx + \int_p^b f(x)\,dx = \int_a^b f(x)\,dx$
   - **Definite integrals: comparison:** If $f(x) \le g(x)$ on the interval $[a, b]$, then $\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$.
2. **Substitution Rule**—a.k.a. ***u*-substitutions:** $\int f(g(x))g'(x)\,dx = \int f(u)\,du$
   - $\int f(g(x))g'(x)\,dx = F(g(x)) + C$ if $\int f(x)\,dx = F(x) + C$.
3. **Integration by Parts**

   Best used to integrate a product when one factor ($u = f(x)$) has a simple derivative and the other factor ($dv = g'(x)\,dx$) is easy to integrate.
   - **Indefinite Integrals:**
   $\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$ or $\int u\,dv = uv - \int v\,du$
   - **Definite Integrals:** $\int_a^b f(x)g'(x)\,dx = f(x)g(x)\big|_a^b - \int_a^b f'(x)g(x)\,dx$
4. **Trigonometric Substitutions:** Used to integrate expressions of the form $\sqrt{\pm a^2 \pm x^2}$.

Expression	Trig substitution	Expression becomes	Range of θ	Pythagorean identity used
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$ $dx = a\cos\theta\,d\theta$	$a\cos\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ $(-a \le x \le a)$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$ $dx = a\sec\theta\tan\theta\,d\theta$	$a\tan\theta$	$0 \le \theta < \frac{\pi}{2}$ $\pi \le \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$
$\sqrt{x^2 + a^2}$	$x = a\tan\theta$ $dx = a\sec^2\theta\,d\theta$	$a\sec\theta$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$

APPLICATIONS

GEOMETRY

Area: $\int_a^b (f(x) - g(x))\,dx$ is the area bounded by $y = f(x)$, $y = g(x)$, $x = a$ and $x = b$ if $f(x) \ge g(x)$ on $[a, b]$.

Volume of revolved solid (disk method): $\pi\int_a^b (f(x))^2\,dx$ is the volume of the solid swept out by the curve $y = f(x)$ as it revolves around the x-axis on the interval $[a, b]$.

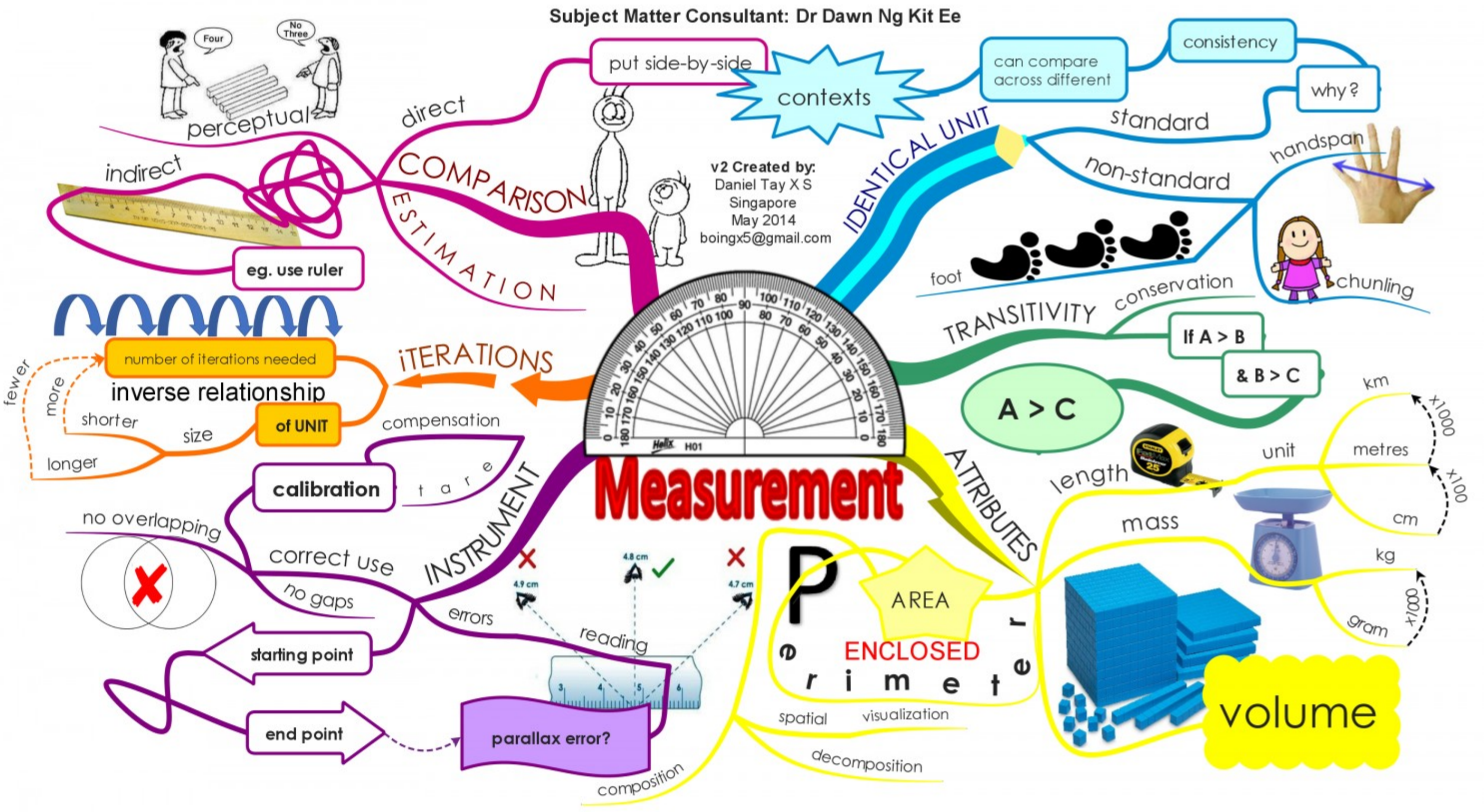
Volume of revolved solid (washer method): $\pi\int_a^b (f(x))^2 - (g(x))^2\,dx$ is the volume of the solid swept out between $y = f(x)$ and $y = g(x)$ as they revolve around the x-axis on the interval $[a, b]$ if $f(x) \ge g(x)$.

Volume of revolved solid (shell method): $\int_a^b 2\pi x f(x)\,dx$ is the volume of the solid obtained by revolving the region under the curve $y = f(x)$ between $x = a$ and $x = b$ around the y-axis.

Arc length: $\int_a^b \sqrt{1 + (f'(x))^2}\,dx$ is the length of the curve $y = f(x)$ from $x = a$ to $x = b$.

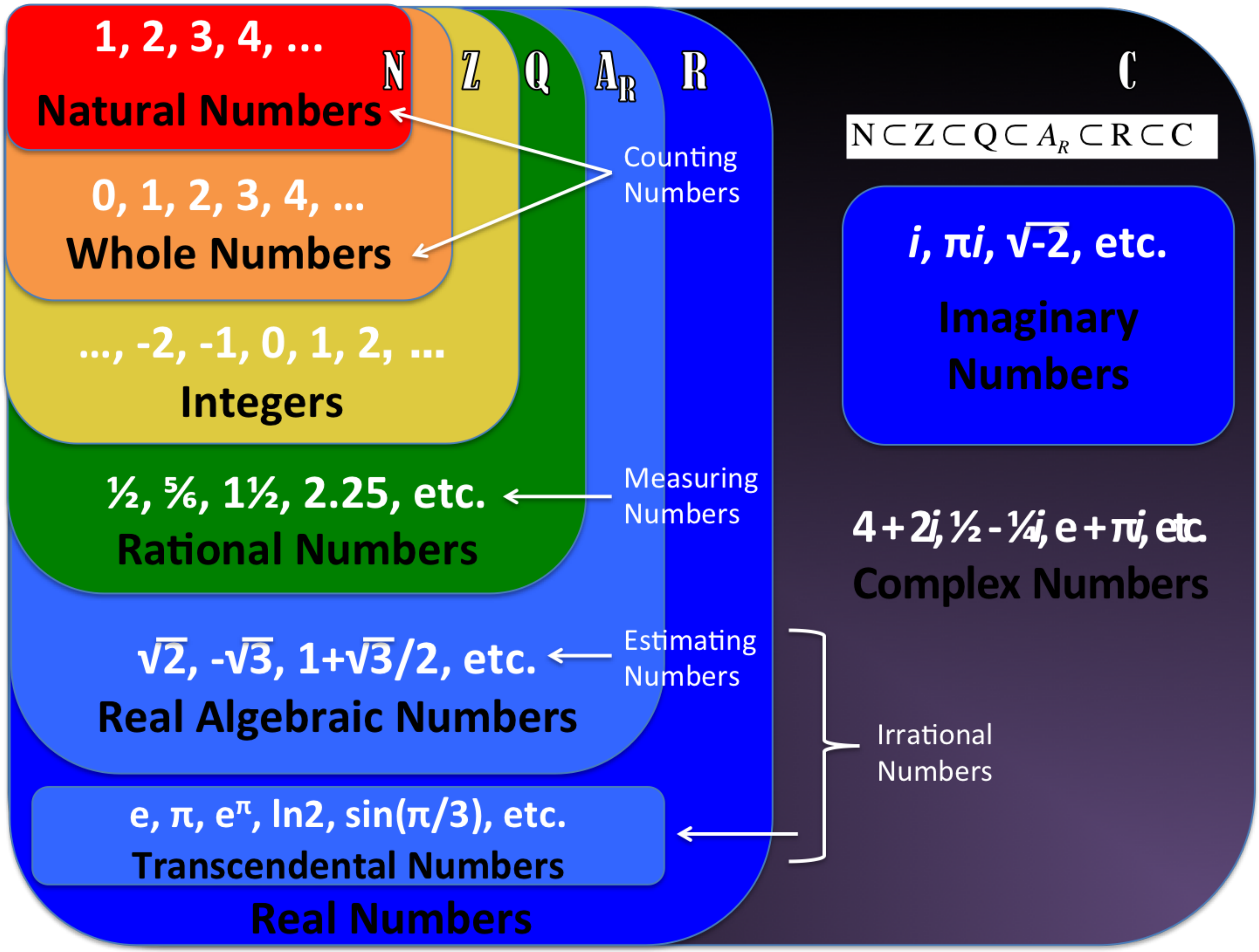
Surface area: $\int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\,dx$ is the area of the surface swept out by revolving the function $y = f(x)$ about the x-axis between $x = a$ and $x = b$.

CONTINUED ON OTHER SIDE

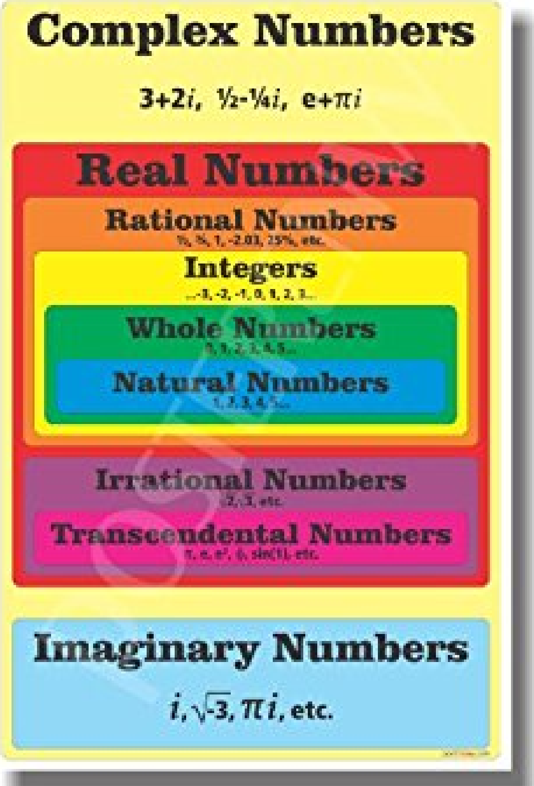


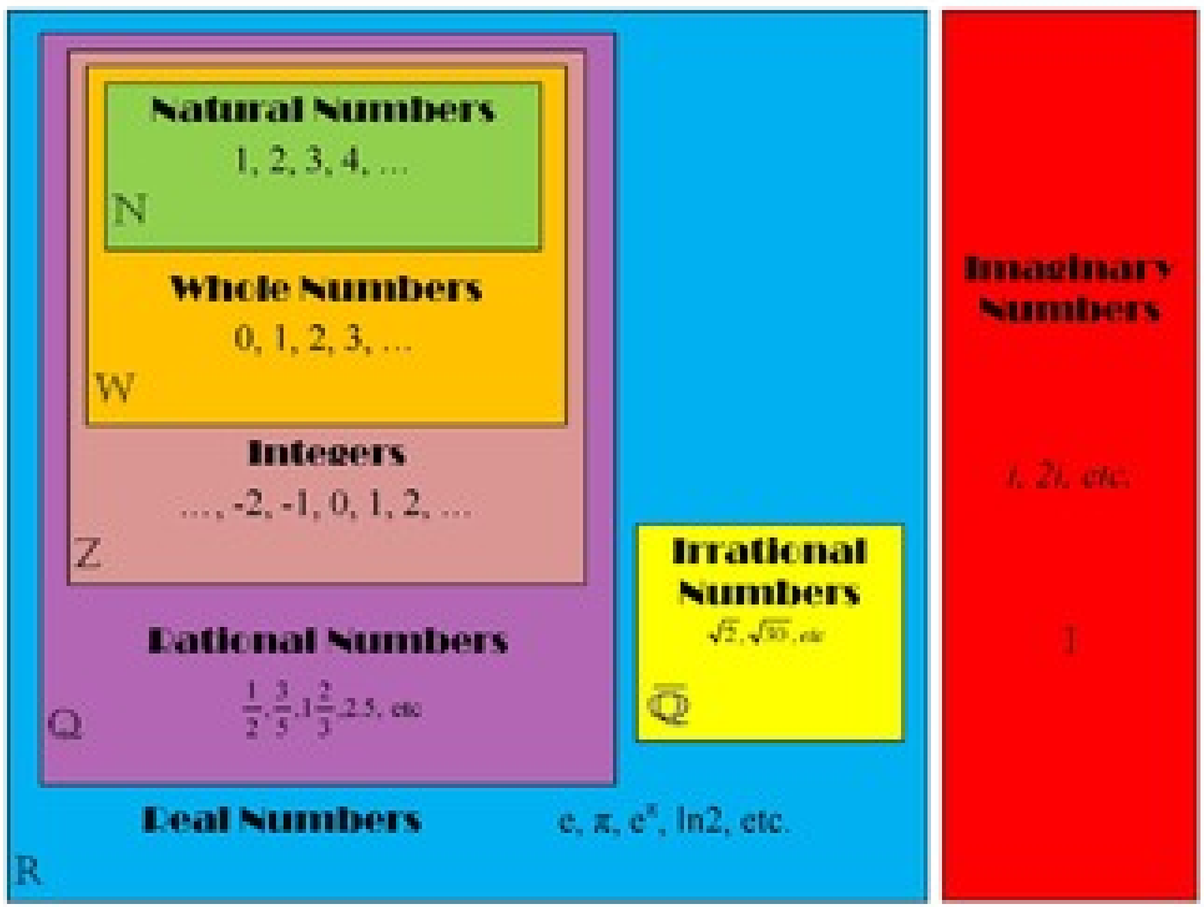
The Greatest Mathematicians of All Time

Created by



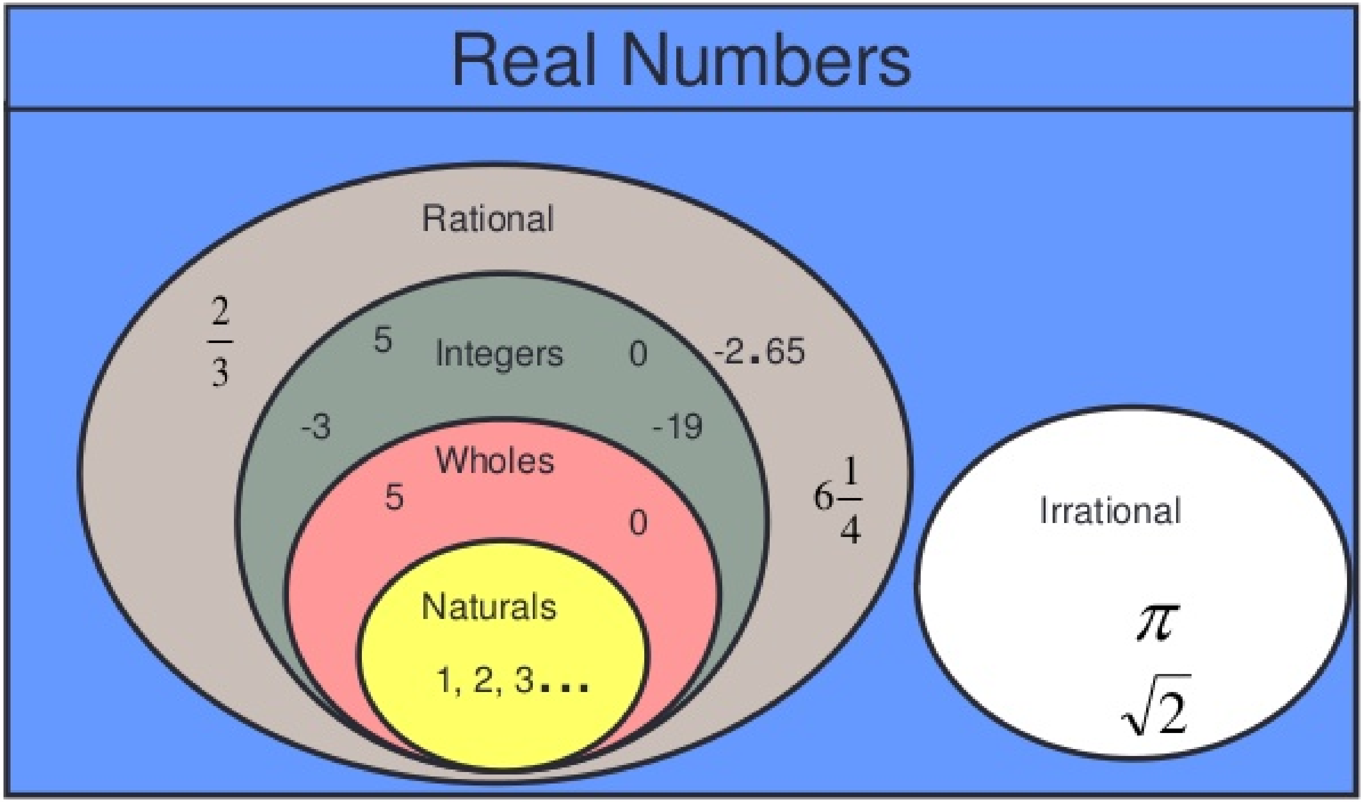
1, 2, 3, 4, ...
Natural Numbers
N
Z
Q
A_R
R
C
$N \subset Z \subset Q \subset A_R \subset R \subset C$
0, 1, 2, 3, 4, ...
Whole Numbers
Counting Numbers
..., -2, -1, 0, 1, 2, ...
Integers
i, πi, √-2, etc.
Imaginary Numbers
½, ⅚, 1½, 2.25, etc.
Rational Numbers
Measuring Numbers
4 + 2i, ½ - ¼i, e + πi, etc.
Complex Numbers
√2, -√3, 1+√3/2, etc.
Real Algebraic Numbers
Estimating Numbers
Irrational Numbers
e, π, e^π, ln2, sin(π/3), etc.
Transcendental Numbers
Real Numbers

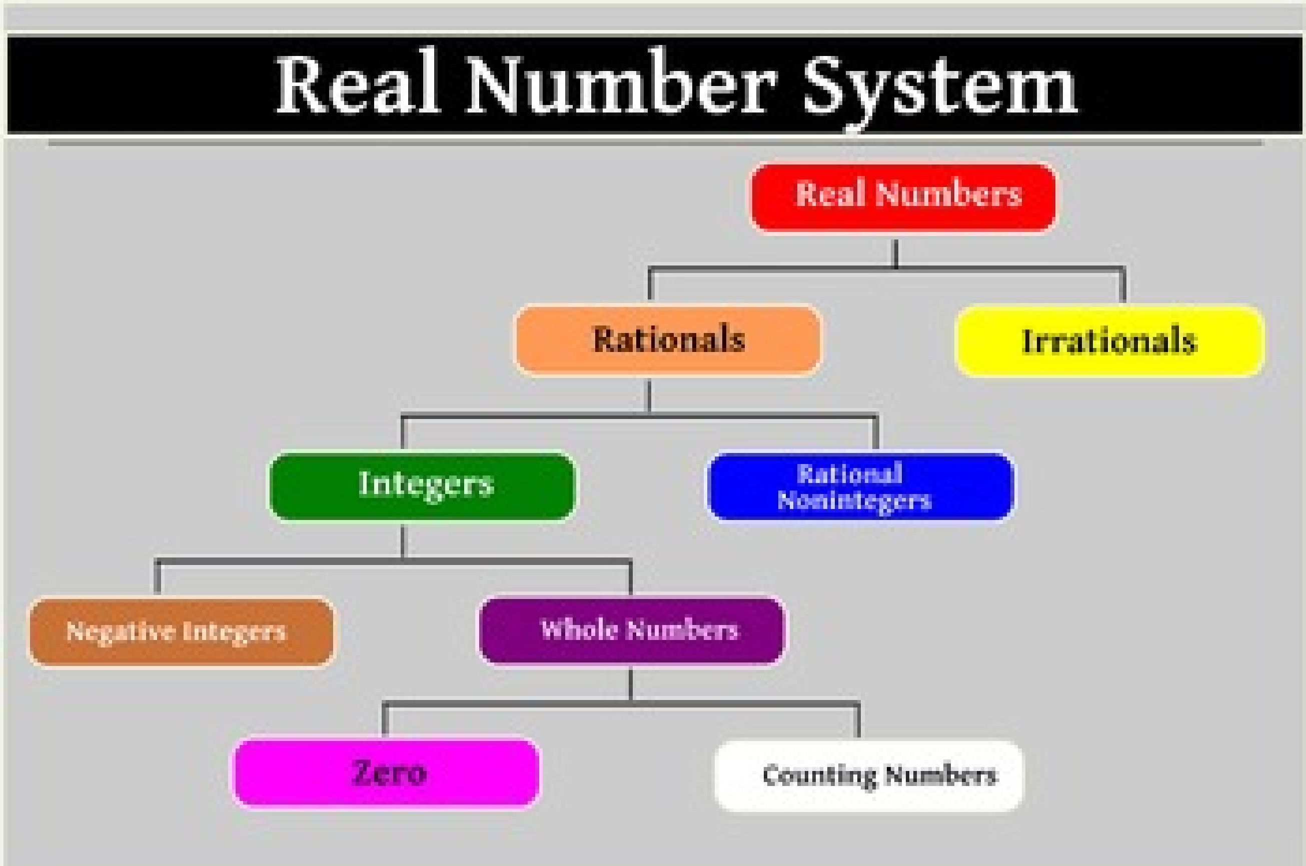

Complex Numbers
3+2i, ½-¼i, e+πi
Real Numbers
Rational Numbers
Integers
...-3, -2, -1, 0, 1, 2, 3...
Whole Numbers
Natural Numbers
Irrational Numbers
Transcendental Numbers
Imaginary Numbers
i, √-3, πi, etc.


Natural Numbers
1, 2, 3, 4, …
N
Whole Numbers
0, 1, 2, 3, …
W
Integers
…, -2, -1, 0, 1, 2, …
Z
Rational Numbers
$\frac{1}{2}, \frac{3}{5}, 1\frac{2}{3}, 2.5$, etc
Q
Irrational Numbers
$\overline{Q}$
Real Numbers
e, π, e^{π}, ln2, etc.
R
Imaginary Numbers
i, 2i, etc.

Real Numbers

This Venn Diagram displays the Sets of Real Number (Rational, Irrational, Integers, Wholes, and Naturals)





Real Number System
Real Numbers
Rationals
Irrationals
Integers
Rational Nonintegers
Negative Integers
Whole Numbers
Zero
Counting Numbers

TYPES OF NUMBERS

NATURAL	(also called counting numbers) are all whole numbers greater than ____________.
WHOLE	________________ numbers starting with zero.
INTEGERS	The set of whole numbers and their ________________.
REAL	The set of numbers that includes all ____________ and ____________ numbers.
RATIONAL	The numbers that can be expressed in the form ________ where a and b are integers and b does not equal zero.
IRRATIONAL	The numbers that cannot be written as a ____________ of two integers.

Basic Properties of Numbers

Commutative

Changing the order of addends or factors does not affect the sum or product.

a + b = c	a x b = c
b + a = c	b x a = c
12 + 6 = 18	5 x 7 = 35
6 + 12 = 18	7 x 5 = 35

Associative

The order in which numbers are grouped does not affect the sum or product.

(a + b) + c = d	(a x b) x c = d
a + (b + c) = d	a x (b x c) = d
(3 + 5) + 2 = 10	(4 x 7) x 3 = 84
3 + (5 + 2) = 10	4 x (7 x 3) = 84

Distributive

Adding two or more numbers together, then multiplying the sum by a factor is equal to multiplying each number alone by the factor first, and then adding the products.

a (b + c) = (a x b) + (a x c)

4 (1 + 8) = (4 x 1) + (4 x 8)

4 x 9 = 4 + 32

36 = 36

Identity

The additive identity is zero. If you add zero to an addend, the sum will equal that addend.	The multiplicative identity is one. If you multiply a factor by one, the product will equal that factor.
a + 0 = a	a x 1 = a
8 + 0 = 8	25 x 1 = 25

or $x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$
y $v_x(t) = v_{0x} + a_x t$
$v_{fx}^2 = v_{0x}^2 + 2a_x \Delta x$

ANGULAR MOMENTUM | $L = \sqrt{l(l+1)}\,\hbar$, $l = 0, 1, 2, \dots n-1$ | $L_z = m_l\hbar$, $m_l = -l \dots 0 \dots l$ | $\theta = \cos^{-1}\left(\frac{L_z}{L}\right) = \cos^{-1}\frac{m_l}{\sqrt{l(l+1)}}$ | $\Delta L_z \cdot \Delta\phi \sim h/2\pi$ z-component

$\vec{\mu} = -\frac{e}{2m_e}\vec{L}$ | $\mu_z = \frac{eh}{4\pi m_e} = \mu_B = 9.274\times10^{-24}\,J/T = 5.788\times10^{-5}\,eV/T$ | $\theta_{min} = \cos^{-1}\frac{2}{\sqrt{2(2+1)}} = \cos^{-1}0.817 = 35.3°$ | Heisenberg's Uncert. Principle

μ = Magnetic Dipole Moment | BOHR MAGNETON | $\mu_z = -m_l\mu_B$ | STERN GERLACH $U(\theta) = -\vec{\mu}\cdot\vec{B}$ $U(\theta) = \mu_z B_z$ | $F_z = \mu_z\frac{dB_z}{dz}$

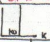
Electron Spin | $S = \sqrt{s(s+1)}\,\hbar$ | $S_z = m_s\hbar$ | $m_s = \pm\frac{1}{2}$ | $\vec{\mu}_s = -(e/m_e)\vec{S}$ | INTRINSIC Angular momentum | Paul Dirac

QUANTUM NUMBERS n Principal Quantum # $n = 1, 2, 3, \dots$ m_l Magnetic Quantum # $m_l = -l, \dots 0 \dots l$ e^-'s
l Orbital Quantum # $l = 0, 1, \dots n-1$ m_s Spin Quantum # $m_s = \pm 1/2$ FERMIONS

$2n^2$ = # OF QUANTUM STATES ORBITAL TYPES (s $l=0$)(p $l=1$)(d $l=2$)(f $l=3$) $2e^-$ PER EACH LOBE. IN GENERAL $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6$ CAPABILITY $n=1\ 2\ 3$ A: s sp spd spdf

PHASE VELOCITY $V_p = \frac{\omega}{K}$

WAVE STUFF | $p = \frac{h}{\lambda}$ | $k = \frac{2\pi}{\lambda}$ | $p = \hbar K$ | $\vec{p} = \hbar\vec{k}$ | Intensity DELTA FUNCTION



FOURIER TRANSFORM $F(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{ikx}dx$ | BREAKING UP A WAVE INTO A BUNCH OF SINE AND COSINE WAVES

No Such Thing as an experimental "pure" sine wave | GROUP VELOCITY | $V_g = \partial\omega/\partial k$ | $\omega = ck$ | $\frac{\partial\omega}{\partial g} = c$ | $V_g = c$ (LIGHT)

GROUND STATE HYDROGEN | $\Psi(r) = \frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$ | $\Psi^2(r) = \frac{1}{\pi a_0^3}e^{-2r/a_0}$ | RADIAL PROBABILITY DENS. $P_r(r) = \Psi^2(r)\,4\pi r^2 = \frac{4}{a_0^3}r^2e^{-2r/a_0}$ | REDUCED MASS CM FRAME | $m = \frac{m_1 m_2}{m_1 + m_2}$ | e^- Fill in order of Increasing Energy

EXCITED STATES OF HYDROGEN | $\Psi_{200}(r) = \frac{1}{4\sqrt{2\pi a_0^3}}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$ | $P_r(r) = \frac{1}{8a_0}\left(\frac{r}{a_0}\right)^2\left(2 - \frac{r}{a_0}\right)^2 e^{-r/a_0}$ | $\Psi_{21}^2(r) = \frac{1}{96\pi a_0^5}r^2e^{-r/a_0}$ | $P_r(r) = \frac{1}{24a_0^5}r^4e^{-r/a_0}$

$eV = h\nu_{max} = hc/\lambda_{min}$ | $\lambda_{min} = hc/eV$ e^- bombardment of materials | $\sqrt{\nu}$

$\nu = \frac{\Delta E}{h} = \frac{m_e e^4 Z^2}{8\epsilon_0^2 h^3}\left(\frac{1}{n^2} - \frac{1}{k^2}\right)$ Let $m=1$, $n=2$, $Z = Z-b$ | $\sqrt{\nu} = \left(\frac{3m_e e^4}{32\epsilon_0^2 h^3}\right)^{\frac{1}{2}}(Z-b)$ $\sqrt{\nu} = aZ - ab$ Kα Transition

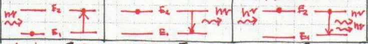
BOHR THEORY AND THE MOSELEY PLOT

PAULI EXCLUSION PRINCIPLE | IN A MULTIELECTRON ATOM, NO TWO ELECTRONS (FERMIONS) CAN HAVE THE SAME 4 QUANTUM NUMBERS (CAN'T OCCUPY THE SAME QUANTUM STATE)

OPTICAL TRANSITIONS SELECTION RULES $\Delta l = \pm 1$ S-P P-S OK S-d d-S NOT OK

LASERS RESONANT CAVITY | MONOCHROMATIC, HIGHLY COHERENT, DIRECTIONAL, SHARPLY FOCUSED
$h\nu = E_2 - E_1$

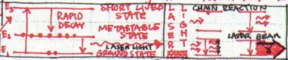
ABSORPTION | SPONTANEOUS EMISSION | STIMULATED EMISSION



UPPER-LOWER STATE # RATIO $\frac{n(E_2)}{n(E_1)} = e^{-\frac{E_2 - E_1}{KT}}$ M-B DIST.

LIGHT AMPLIFICATION by STIMULATED EMISSION OF RADIATION L.A.S.E.R.

POPULATION INVERSION (OPTICAL PUMPING)



RADIOACTIVE DECAY | $\frac{N(t)}{N(0)} = e^{-t/\tau} = e^{-pt}$ | p = probability, τ = lifetime
$t_{\frac{1}{2}} = \tau\ln 2$ HALF LIFE | β decay $n \rightarrow p + e^- + \bar{\nu}$

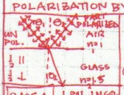
ELECTRONS ARE INDISTINGUISHABLE (Pauli Excl. CAVEAT) FOR A SYSTEM OF FERMIONS THE Ψ'S MUST BE (ANTISYMMETRIC) CHANGING THE ORDER OF ITS PAIRS CHANGES THE SIGN OF Ψ
N = # OF ELECTRONS
α SPIN UP $+\frac{1}{2}$ β SPIN DOWN $-\frac{1}{2}$ $|\Psi|^2$ IS INDEPENDENT OF PARTICLE LABEL, BUT Ψ IS NOT $\pm$ SIGN

$\Psi_{He} = \frac{1}{\sqrt{2!}}\begin{vmatrix} 1s\alpha(1) & 1s\beta(1) \\ 1s\alpha(2) & 1s\beta(2) \end{vmatrix}$ | $\Psi_x = \frac{1}{\sqrt{N!}}$ $|1s\alpha(1) \dots$ N×N Matrix | DETERMINANT ENSURES THAT THE WAVE FUNCTION IS ANTI-SYMMETRIC | $\Psi_H = R(r)\,\Theta(\theta)\,\Phi(\phi)$ (r, θ, ϕ) SPHERICALLY SYMMETRIC

BINOMIAL EXPANSION (APPROX.) $(1+x)^K \approx 1 + Kx + \frac{K(K-1)x^2}{2!}$

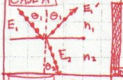
COMPOUND LENS/MIRROR SYSTEMS | Use the image from the 1st system as the OBJECT for the 2nd system. $\boxed{\text{Let } O_2 = -(i_1 - d)}$ d = dist between pieces. For whole effective focal Length, use
$$\frac{1}{O_{INITIAL}} + \frac{1}{i_{FINAL}} = \frac{1}{f_{SYSTEM}}$$

POLARIZATION BY REFLECTION



UNPOLARIZED LIGHT IS A SUPERPOSITION OF CASE A AND CASE B
If $\theta_1 = \theta_B$ (BREWSTER'S ANGLE)
CASE A E_1' GETS THROUGH
CASE B $E_1' = 0 \Rightarrow$

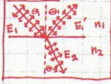
CASE A ⊥ POL. INCD.



$n_2\cos\theta_1 = n_1\cos\theta_2$
$n_1\sin\theta_1 = n_2\sin\theta_2$ (Snell)
Can Derive $\theta_B + \theta_2 = 90$
$\tan\theta_B = \frac{n_2}{n_1}$ $I = E^2$

INCIDENT UNPOLARIZED LIGHT

CASE B || POL. INCD.



$\frac{E_2}{E_1} = \frac{2n_1\cos\theta_1}{n_1\cos\theta_2 + n_2\cos\theta_1}$ (REFRACTED)

$\frac{E_1'}{E_1} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_1\cos\theta_2 + n_2\cos\theta_1}$ (REFLECTED)

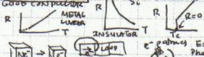
PAINT DROP PROBLEM Approx $\Delta p\Delta x_0 = \frac{h}{2\pi}$

? Δx min

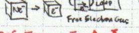
$\Delta x = \Delta x_0 + \Delta v\,t$ $\Delta y = H - \frac{1}{2}gt^2$ $\Delta y = 0 \Rightarrow t = \sqrt{\frac{2H}{g}}$ $\Delta p = m\Delta v$
$\Delta x = \Delta x_0 + \frac{\Delta p}{m}t = \Delta x_0 + \frac{h}{2\pi\Delta x_0 m}t$ MINIMIZE Δx SET $\frac{d\Delta x}{d\Delta x_0} = 0$ FIND Δx_0 min Plug Back into Δx Equation

SIMPLE HARMONIC OSCILLATOR | Ground St. Approx $px = \frac{h}{2\pi}$ $\Delta x \approx x$ $\Delta p \approx p$
(Ground St. Energy) $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \frac{p^2}{2m} = \frac{1}{2}kx^2 + \frac{h^2}{4\pi^2x^2}\frac{1}{(2m)}$ MINIMIZE E
Set $\frac{dE}{dx} = 0$ solve for x Plug into E $E = \frac{3}{2}\hbar\sqrt{\frac{k}{m}} = \frac{3}{2}\hbar\omega$ Real Answer $= \frac{1}{2}\hbar\omega$ Grade | $\Delta x = \left(\frac{h}{2\pi m}\right)^{\frac{1}{2}}\left(\frac{H}{g}\right)^{\frac{1}{4}}$

SOLID STATE | GOOD CONDUCTOR | SEMICONDUCTOR | SUPER CONDUCTOR



$n_q = N/V$ # density
$T_c \approx 100K$ Record Today
$J = n_q q V_d$ $R = \frac{\rho L}{A}$ $J = \sigma E$ Steady State
$\boxed{\sigma = \frac{n_q q^2\tau}{m}}$ τ = damping time Classical understanding



e^- pairing

$E = \hbar\omega$ PHONONS $\omega = \sqrt{\frac{k}{m}}$ | m-∂'s Isotopes | BCS THEORY $\boxed{T_c = \omega e^{-\frac{1}{\lambda - u}}}$ Pairing E $2\Delta = 3.5kT_c$
WORKS FOR LOW TEMP SUPER C.
BCS Increase T_c $\uparrow\omega$ $\Downarrow m$ $\lambda = e^-$ phonon interaction $u = e^- - e^-$ repulsion $P = F/A$ $\uparrow P$ $\uparrow F$

$\sigma \sim e^{-E_g/KT}$ $\rho \sim e^{E_g/KT}$ $C_V = \frac{1}{2}\pi^2 NK\frac{T}{E_F}$
E_F = Fermi Energy
E of Highest Occupied St.

sides are themselves equal

Useful Math Hacks

Multiplying Large Numbers Mentally

97 X 96 = 93 12

100 − 97   100 − 96   100 − 7

3 + 4 = 7

X

Multiplication by 11

Eg) 32 X 11

Step 1	Step 2
32	3 + 2
3 2	3 5 2

You can remember the value of 'Pi' by counting each word's letters in "May I have a large container of Coffee"

Nine Times Table

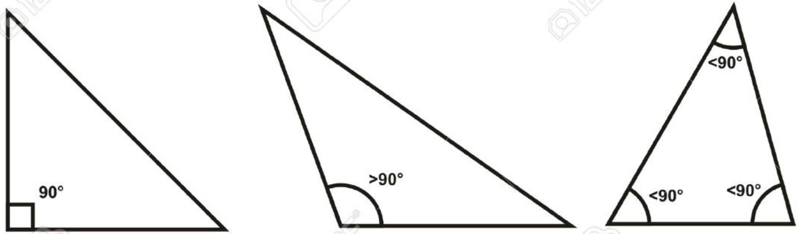
9 x 1 =	09
9 x 2 =	18
9 x 3 =	27
9 x 4 =	36
9 x 5 =	45
9 x 6 =	54
9 x 7 =	63
9 x 8 =	72
9 x 9 =	81
9 x10=	90

Left side runs from 0 to 9.

Right side runs from 9 to 0.

House Tutor - Home Tutor Singapore

Types of Triangle



Right Triangle

Right Triangle is a triangle in which one angle is equal to 90 degree angle

Obtuse Triangle

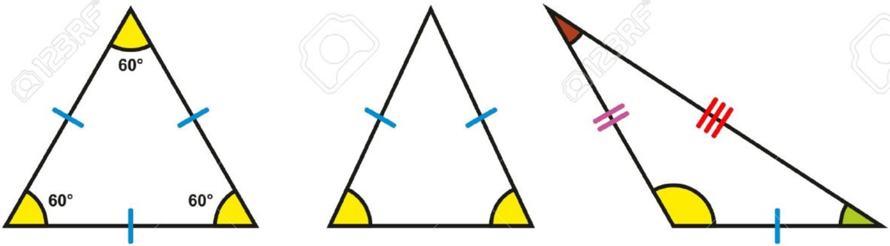
Obtuse Triangle is a triangle with one obtuse angle (greater than 90 degree) and two acute angles

Acute Triangle

Acute Triangle is a triangle with all three angles acute (less than 90 degree)

Oblique Triangles

Oblique Triangle is a triangle that is not right triangle, because it has no 90 degree angle



Equilateral Triangle

Isosceles Triangle is a triangle that has two sides of equal length, and the two angles opposite the equal sides are themselves equal

Isosceles Triangle

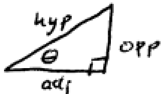
Equilateral Triangle is a triangle in which all three sides are equal and all three angles are each 60 degree

Scalene Triangle

Scalene Triangle is a triangle that has all its sides of different lengths and all angles of different measures

TRIG REVIEW Sheet

1. Right Triangles:

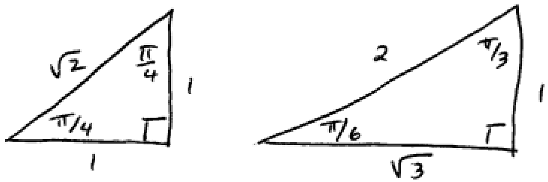


$\sin\theta = \frac{opp}{hyp}$ $\qquad \csc\theta = \frac{hyp}{opp} = \frac{1}{\sin\theta}$

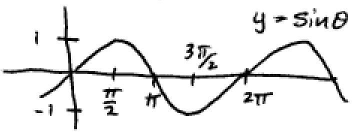
$\cos\theta = \frac{adj}{hyp}$ $\qquad \sec\theta = \frac{hyp}{adj} = \frac{1}{\cos\theta}$

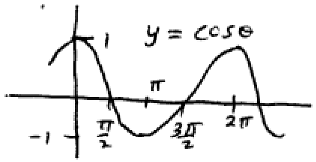
$\tan\theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \frac{adj}{opp} = \frac{\cos\theta}{\sin\theta}$

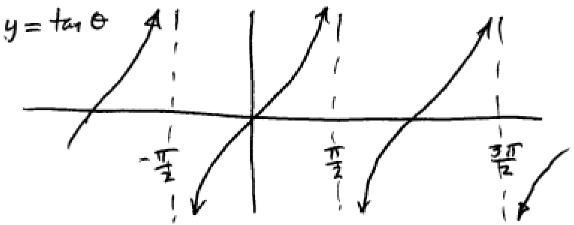
2. Special Triangles & Ratios.



3. Graphs

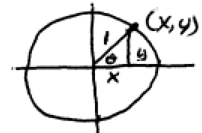




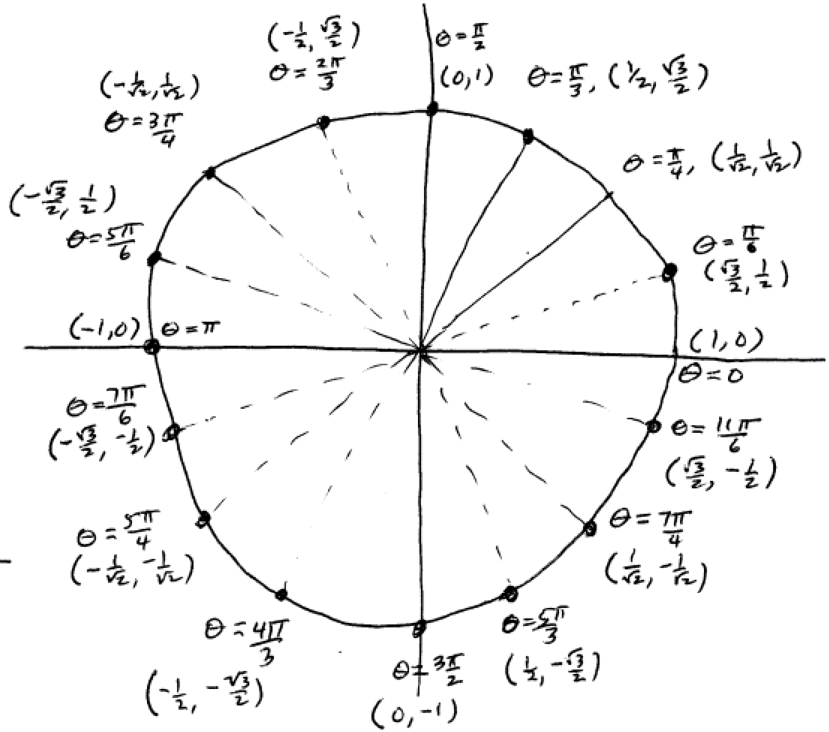


4. The Unit Circle

$(x, y) = (\cos\theta, \sin\theta)$



use this to determine $\cos\theta$, $\sin\theta$ for lots of different angles:



$., + - \pm \infty = \neq \sim \times \cdot \div ! \alpha < \ll > \gg \leq \geq \mp \rightarrow - \sqrt[3]{} \sqrt[4]{}$

$\alpha\beta\gamma\delta\partial\varepsilon\epsilon\theta\vartheta\mu\pi\rho\sigma\tau\varphi\omega \quad \frac{dy}{dx} \frac{\Delta y}{\Delta x} \frac{\partial y}{\partial x} \frac{\delta y}{\delta x} \iiint \oiiint$

$\sum \lim_{n \to \infty} (\,) (\,) [\,] \{\,\} f(x) |x| \Delta \sin \cos \tan \cot \sec \csc$

$\log \ln \min \max$

ABCDEFGHIJKLMN
OPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz
abcdefghijklmnopqrstuvwxyz
abcdefghijklmnopqrstuvwxyz

1234567890
1234567890
1234567890

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$ax^2 + xb + c = 0$$

$$\log_b mn = \log_b m + \log_b n$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

$$\int \frac{x\,dx}{1 + \sin ax} = -\frac{x}{a} \tan\left(\frac{\pi}{4} - \frac{ax}{2}\right) + \frac{2}{a^2} \ln \cos\left(\frac{\pi}{4} - \frac{ax}{2}\right) + C$$